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Reliability-based evaluation of the prestress level in concrete containments with unbonded tendons

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Abstract

It has been a common practice both in Sweden and worldwide to enclose nuclear reactors with prestressed concrete structures. The prestress level decreases with time from its initial value due to various degradation mechanisms. To ensure that the prestress level is sufficient the tendon force is measured at regular in-service inspections. The intention with this paper is to present a reliability-based procedure to evaluate the prestress level on the basis of data from in-service inspections. Existing approaches to evaluate the prestress level do not take into account the variability in the measured prestress. It is not possible to achieve a complete assurance concerning the prestress level. However, by using a probabilistic model, involving the variability in the measuring result and the structural behaviour, the prestress level could be confirmed in a more stringent way. Both the time dependent loss of prestress and the possibility of tendons being broken (due to defects as corrosion) are considered. To avoid through-wall cracks in the concrete it is required that the prestress shall counterbalances the tensile stresses expected at an internal accident. The factor of interest is the prestress level in the concrete and not the force in individual tendons. Several tendons influence the prestress level in a specific part of the containment. The required prestress level shall be fulfilled in all parts of the containment where each part is influenced by a number of individual tendons. It is suggested in this paper that this problem can be analysed as a structural reliability problem idealized as a series of correlated parallel subsystems.

Keywords: Nuclear; Loss of prestress; Tendon; Parallel system; Series system; Resistance deterioration

1. Introduction

In Sweden and worldwide it has been a common practice to enclose nuclear reactors with prestressed concrete containments. These containments shall work as safety barriers between the reactor and the surrounding environment. The typical layout for Swedish reactor containments is shown in Fig. 1 and the main design criterion is to preserve tightness at the overpressure that will occur in the event of a major internal accident. The prestressing system plays a major role maintaining a high structural integrity of the reactor containment. To

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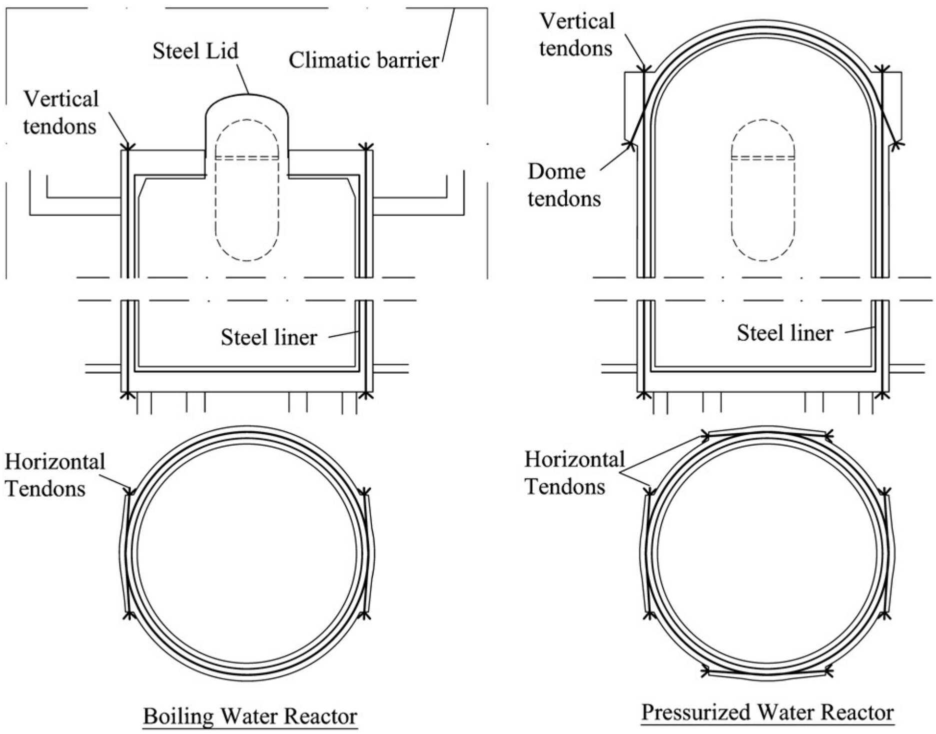

Fig. 1. Principal layouts of Swedish containments.

avoid through-wall cracks in the concrete the prestress shall counterbalance the tensile stresses expected during an internal accident.

The prestress will gradually decrease from its initial value due to time dependent deformation (longtime loss). Long-time losses arises from shrinkage and creep in the concrete and relaxation in the tendons. These mechanisms depend on several different environmental and material factors which make the loss difficult to predict. In design, the long-time loss for the theoretical lifetime of the structure (around 30 years) is generally assumed to be around 20% of the initial prestress (see Anderson [1]). Another factor which could affect the level of prestress in the concrete structure is tendon corrosion (causing defective tendons). The most common method to protect tendons from corrosion, for prestressed structures in general, is to inject the space between tendon and duct with cement grout (bonded tendons). However, in reactor containments it is often required to use unbonded tendons (i.e. not injected with cement grout) to be able to inspect the tendons. Six of the twelve Swedish reactor containments are constructed with unbonded tendons. Instead of cement grout the unbonded tendons are protected from corrosion with injection of grease or by ventilation of dry air.

For containments with unbonded tendons regular in-service inspections are made to estimate the prestress level in the structure and to identify the condition of the tendons (see Anderson et al. [2]). In Sweden these in-service inspections are made according to an American guide [3]. This guide prescribes that the tendon force shall be measured with so-called lift-off technique or other equivalent method testing prestress. In methods using lift-off technique a calibrated jack is used to find the force at the tendon end. Contractors performing lift-off tests in Sweden estimate an error for the measured force of ±2%. Apart from measuring the tendon force the guide prescribes that an in-service inspection should consist of a visual check of the concrete surrounding the tendon anchor. One wire in two of the tendons shall also be removed and checked over its whole length to observe corrosion or other material defects. Finally, the condition of the medium for corrosion protection is verified. The inspections shall, according to the guide, be performed 1, 3, 5 years after the structural integrity test and thereafter every 5th year. The post-tensioned system in reactor containments consists of hundreds of tendons, so it is not economically feasible to test all tendons. The guide recommends that between 2 and 4% of the tendons should be selected randomly at each inspection.

In Regulatory Guide 1.35 [3], the evaluation of the prestress level is made according to two different approaches. (1) Each measured tendon force is compared with a lower limit for the *predicted force* at the time of the inspection. (2) The mean value of the measured tendon forces in one group (i.e. vertical tendons or horizontal tendons, see Fig. 1) is compared with the *minimum required prestress level* from design.

Existing approaches to evaluate the prestress level do not take into account the variability in the measured prestress. It is not possible to achieve a complete assurance concerning the prestress level. However, by using a statistical model, involving the variability in the measuring result and the structural behaviour, the prestress level could be confirmed in a more stringent way. The intention with this paper is to present a reliability-based method to decide if the required prestress level is achieved on the basis of results from in-service inspections. Both the general loss of force (due to time dependent material deformation) and the possibility of one or several tendons being broken (due to corrosion or other material defects) are considered.

To fulfill the requirement of tightness at the internal design pressure the factor of interest is the prestress level in the concrete and not the force in individual tendons. Several tendons influence the prestress level in a specific part of the containment. The required prestress level shall be fulfilled in all parts of the containment where each part is influenced by a number of individual tendons. It is suggested in this paper that this problem can be analysed as a structural reliability problem idealized as a series of correlated parallel subsystems.

A reliability based evaluation of prestressed containments with bonded tendons was made by Pandey [4]. Bonded tendons are not accessible for direct inspections as unbonded tendons are. The evaluation method in Pandey [4] was therefore based on measurements on test beams with similar conditions as the containment wall.

2. Outline of Swedish reactor containments

All the Swedish reactor containments are constructed as concrete cylinders founded on thick concrete plates. The top of the cylinders is either enclosed with a massive steel lid or with a prestressed concrete dome. The cylinder wall consists of an external bearing concrete shell, which is 0.76–1.2 m thick and post-tensioned in two directions (vertical and horizontal). Inside the concrete shell a 5–10 mm thick steel liner secures the tightness. The inside of the steel liner is protected from missiles (e.g. from pipe pieces) by a 0.26–0.33 m thick reinforced concrete shell. Fig. 1 shows the principal outline of the containments for the two types of reactors used in Sweden. It should be noted that all the containments have a large number of penetrations and other discontinuities not shown in the figure. For more detailed description of the layout of reactor containments, see Nuclear containments [5].

Two different types of tendon systems are used for reactor containments in Sweden, BBRV- and VSL-systems (see Fig. 2). The tendon in BBRV-systems consists of a large number of single wires with fixed length. In this system, the tendons are tensioned by pulling the anchor head backwards and is fixed in the right position with shim plates between the bearing plate and the anchor head. VSL-tendons consist of a number of strands, which are tensioned by pulling directly in the strands. The strands are fixed with wedges in the anchor head, which has direct contact with the bearing plate. Typical tendons used in Swedish containments are BBRV tendons with 139 wires with diameter of 6 mm (ultimate load $F_u = 7.1$ MN) and VSL tendons with 19 strands consisting of 7 wires each (ultimate load $F_u = 3.5$ MN).

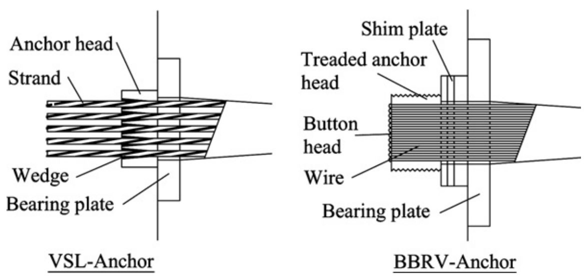

Fig. 2. Principal section of BBRV and VSL anchorages.

3. Structural requirements

The prestress level is designed to prevent failure in the structural parts of the containment. Swedish containments are designed for an overpressure of around 0.5 MPa (internal design pressure p_{d}). According to Swedish requirements the concrete must be in compression state at the internal design pressure, i.e. tensile stresses resulting from the internal design pressure shall not exceed the prestress. This so-called limit state of decompression can be seen as a conservative requirement, since the steel liner will maintain tightness even for loads that gives tensile stresses in the concrete. If the design pressure is increased by 50% the Swedish requirements also prescribe that the tensile yield limit for structural steel, reinforcement and tendons shall not be exceeded. This requirement will in general result in a lower allowable limit for the prestress, due to the generally high percentage of reinforcement in the containment. The main requirement for the prestress level, which refer to the limit state of decompression, can be expressed by the limit state function

$$q(\mathbf{X}) = q_R - q_s \tag{1}$$

where q_R is the prestress and q_S is the tensile stress from the design pressure. X is a vector of random variables, $q(X) > 0$ defines acceptance and $q(X) \leqslant 0$ a violation of the requirements. The event of violation of the requirement $q(X) \leqslant 0$ (tension stress in the concrete) will further in this paper be denoted as event E. The prestress q_R is a random variable to be estimated from measured tendon forces.

The tensile stress from the design pressure q_S is determined from the internal design pressure p_{d}. The level as well as the variability of the design pressure are difficult to evaluate and is based on both expert opinions and specific studies. In Ellingwood [6], different design parameters important to the containment safety is discussed. The peak internal pressure load in Ellingwood [6] is assumed to be described by Type 1 distributions with a coefficient of variation (COV) of around 0.2. As mentioned before the limit state of decompression is a conservative requirement due to the presence of steel liner in Swedish containments. In this paper, the required prestress q_S is therefore seen as a required deterministic limit and not as a random variable. In Sweden, all containments are provided with accident-valves which will depressurize the containment thought filters if the internal pressure exceeds the design pressure p_{d}. The presence of accident-valves also speaks for using a deterministic load limit. However, the reliability model presented in Section 5 can be extended to reflect any variability of the applied load.

In the containment wall the dead load will increase the compression load in the vertical direction. This additional load is however less than 10% of the prestress and is normally neglected when the prestress is evaluated.

The probability of a major internal accident is in some literature specified to 10^{-3} per year (see for example Pandey [4]). It is usually assumed that the reactor is safe if the probability of leakage to the environment is less than 10^{-7} per year (see Nuclear containments [5]). The conditional probability of failure for the containment structure in the event of an internal accident should therefore be less than 10^{-4}.

4. Measuring results of forces in tendons

At each in-service inspection around 4% of the tendons is tested, which corresponds to about 6 vertical and 10 horizontal tendons. Details about the measuring results achieved from all Swedish in-service inspections are thoroughly described in Anderson [1].

The variability in measured tendon force varies significantly between different in-service inspections. The evaluated COV:s of measured tendon forces from all Swedish in-service inspections are in the region of 0.01–0.08. Fig. 3 shows the evaluated COV from each in-service inspection. The dashed lines in the diagram show the least square estimation (LSE) of the evaluated COV for the two different tendon systems (VSL and BBRV). VSL tendons show a higher variability than BBRV tendons. COV for VSL tendons is generally twice as high as for BBRV tendons. The COV obtained for measurements performed directly after the initial tensioning (measurements made on all tendons in the containments) is around 0.015 for containments with BBRV tendons and around 0.03 for containments with VSL tendons. A tendency seen in Fig. 3 is that the COV is increasing with time, which can be expected due to the variability in the mechanisms of long time losses. This tendency is clearly shown for BBRV tendons. For VSL tendons the increase of COV is shown

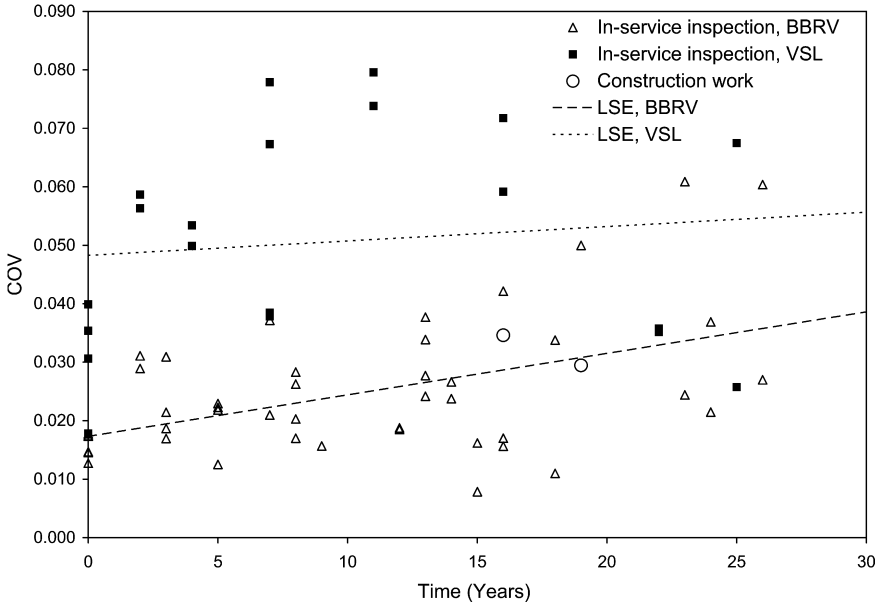

Fig. 3. COV dependency with time. Dots show evaluated COV from all Swedish in-service inspection. Dashed lines show the LSE for each tendon system (BBRV and VSL). Circles show evaluated COV from construction work at R2 and R3.

to be less significant. However, the tendency for VSL tendons is based on less number of in-service inspections and is therefore not as well-founded as the tendency for BBRV tendons (see Fig. 3).

A large number of tendons were tested at two of the Swedish containments, Ringhals 2 (R2) and Ringhals 3 (R3) in connection with large construction work (exchange of steam generator). This construction work was made 16 and 19 years after the initial tensioning and included more than 150 measurements for each containment. Both the containments have BBRV tendons and the COV:s evaluated from these measurements was 0.035 for R2 and 0.029 for R3. These values show good agreement with the LSE for BBRV tendons in Fig. 3.

An assumption of normal distributed tendon force is often made (see for example Ellingwood [6]). Fig. 4 shows normal probability plots for measured tendon forces at Ringhals 2 and 3 directly after the initial

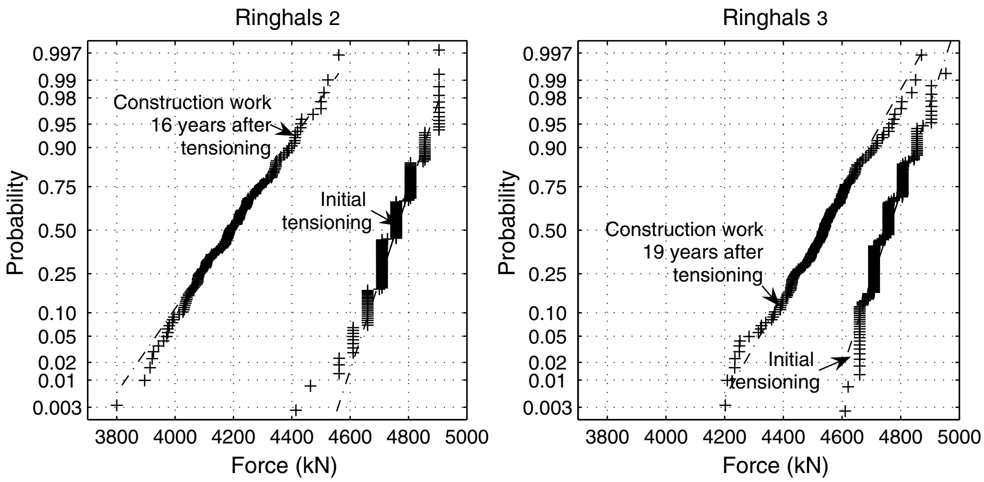

Fig. 4. Measured tendon forces from Ringhals 2 and 3 in normal probability plots.

Table 1
Kolmogorov–Smirnov test for measuring results from Ringhals 2 and 3 directly after the initial tensioning and at the construction work 16 and 19 years later ($\alpha = 0.05$)

	Distribution	P-value	Hypothesis H_0
Ringhals 2 $t = 0$	$N(4748, 82.29)$	0.00125	Rejected
Ringhals 2 $t = 16$	$N(4195, 145.2)$	0.945	Not rejected
Ringhals 3 $t = 0$	$N(4755, 69.19)$	1.95×10^{-4}	Rejected
Ringhals 3 $t = 19$	$N(4535, 133.6)$	0.879	Not rejected

tensioning and at the construction work described above. Initially (1973 for R2 and 1976 for R3) the data seem to be grouped at specific force levels. The reason for this is probably the rough measuring scale used at that time. This phenomenon is not found in the data observed at the construction work 16 and 19 years later.

The data shown in Fig. 4 seem to fit the assumed normal distribution quite well. To evaluate the goodness of fit, the Kolmogorov–Smirnov test is used. The hypothesis, H_0, for this test is that the data are distributed according to a normal distribution. The test is performed at a significance level of 0.05 and the results are shown in Table 1. The measuring data at the construction work show good fit with the normal distribution. The lack of fit initially is assumed to depend on the rough measuring scale mentioned above.

Another factor that speaks for using the normal distribution in this paper is the fact that the prestress level in the concrete, which is of interest, is a sum of a number of influencing tendons. Even if the force in the tendons differs from the normal distribution the level of prestress in the structure will be close to normally distributed due to the central limit theorem.

The tendon forces are here assumed to be independent random variables. The containment concrete wall is thick and very stiff compared to the steel tendons. This fact implies that a change of force in a specific tendon will have little influence on the force in any other tendon in the structure. However, some dependence can be expected due to locally high temperature (increased long time losses), variation of concrete quality (within batches) and elastic shortening (from the initial tensioning).

5. Reliability model

The model described in this section intends to evaluate the probability of event E i.e. the probability that the prestress being lower than the required prestress. The needed input for this model is the tendon mean force μ_F and a standard deviation σ_F valid at the time of the in-service inspection. The sample mean from one in-service inspection is assumed to be a good estimate of the mean force μ_F. According to Section 4 the estimated variability from a single in-service inspection is unreliable. It is therefore preferred to evaluate the standard deviation σ_F from trends over time instead of estimations from a single in-service inspection. It is assumed that the force F in any tendon (similar to the tested) is an independent and normally distributed random variable (see Section 4).

$$F \in N(\mu_F, \sigma_F) \tag{2}$$

The prestress in any point and direction of the containment structure depends on the actual force in a number of influencing tendons. The structure can be assumed to be in compressive state (according to the limit state of decompression) and below the concrete compression strength. Linear elastic theory can therefore be used to express the influence of tendon force on the concrete structure. The limit state of decompression shall be fulfilled in all locations $L(x, y)$ of the containments structure (constant stress in the direction of the wall thickness is assumed). With the knowledge of the structural behaviour a finite number of positions L_j can be defined. Each location L_j represents a specific structural part A_j. The prestress in all positions $L(x, y)$ within the structural part A_j is assumed to be represented by position L_j.

5.1. Model for one structural part

The prestress $q_{R,j}$ representing the structural part A_j can be calculated as

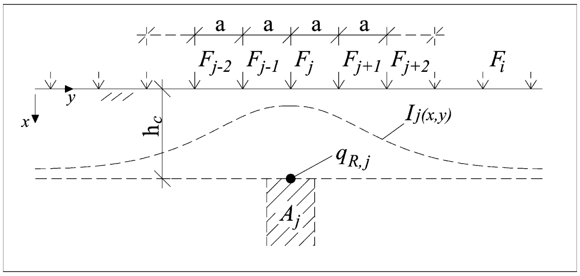

Fig. 5. Special case of vertical tendons influencing the structural element A_j.

$$q_{R,j} = \sum_{i=1}^{N} F_i I_{i,j} \tag{3}$$

where N is the total number of tendons, F_i is the force in tendon i and $I_{i,j}$ is the influence factor describing the effect of tendon i on the structural part A_j. Fig. 5 illustrates the influence for vertical tendons in the containment wall. Since the height of the containment is much larger than the distance between tendons (a) the theory of a semi-infinite elastic shell can be used to model the influence. The prestress is calculated at a certain distance h_c from the edge which must be determined in accordance with the detailed design of the structure. The distance h_c can for instance be taken as the thickness of the top or bottom slabs of the containment. The specific case of prestress from vertical tendons in the containment wall is discussed in the example in Section 7.

In the case of vertical tendons, the ducts are assumed to be more or less straight and the influence of friction along the tendons is therefore small. Horizontal tendons in the containment wall are curved and have a significant friction which means that the force will change along the tendon. This fact has to be considered when the horizontal prestress in the containment wall is evaluated. The influence of friction could be taken into account by the influence factor I.

The tendons influencing the structural part A_j can be seen as a parallel system. Assuming that the tendon forces are independent and normally distributed the prestress $q_{R,j}$ will also be normally distributed with a mean $\mu_{q,j}$ and standard deviation $\sigma_{q,j}$ given by

$$\mu_{q,j} = \mu_F \sum_{i=1}^{N} I_{i,j} \tag{4}$$

$$\sigma_{q,j} = \sigma_F \sqrt{\sum_{i=1}^{N} I_{i,j}^2} \tag{5}$$

The stress influence from the tendons represents the resistance and the tension stress in the wall from internal design pressure represents the load effect in the model. The probability of violation $P(E_j)$ for the structural part A_j is given by

$$P(E_j) = \Phi\left(\frac{\mu_{q,j} - q_S}{\sigma_{q,j}}\right) \tag{6}$$

where Φ is the normal distribution function.

5.2. Model for the whole structure

According to the limit state of decompression the whole containment structure shall be in a compression state at a major internal accident. The requirement shall be fulfilled in all structural parts A_j, i.e. if the prestress is below the required stress in any section a violation of the requirements occurs. This means that the reliability model for the whole structure can be described as a series system of a number of the parallel systems (see Fig. 6). The parallel system j represents the prestress in one of totally n structural parts A_j.

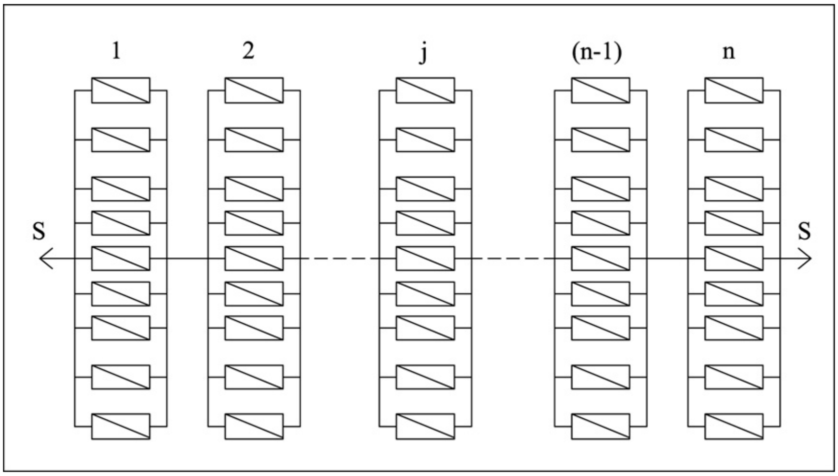

Fig. 6. Series system representing the reliability of the whole structure.

The prestress in adjacent structural parts are influenced by a number of the same individual tendons, i.e. the structural parts in the series system have some degree of correlation. The correlation between different structural parts varies depending on the distance between them. This type of unequally correlated series system requires extensive calculations to find the exact probability of violation (see Thoft-Christensen and Baker [7]). An upper bound probability of violation for the whole structure can be found if it is assumed that the elements in the series system are independent. The upper bound expression can be written as

$$P(E) = 1 - \prod_{j=1}^{n}(1 - P(E_j)) \tag{7}$$

where event E is the event that the limit state of decompression is violated in one or more structural parts. A fully correlated series system gives a lower bound probability of violation and is described in expression.

$$P(E) = \max_{j=1}^{n}(P(E_j)) \tag{8}$$

More narrow limits can be found by using Ditlevsen bounds.

The mean correlation which is shown in expression (9) could be used to evaluate the effect of correlation (see Thoft-Christensen and Baker [7]).

$$\bar{\rho} = \frac{1}{n(n-1)} \sum_{\substack{u,v=1 \\ u \neq v}}^{n} \rho_{u,v} \tag{9}$$

where $\rho_{u,v}$ represents the correlation between the structural elements u and v. Each correlation parameter can be calculated by expression (10) where I is the influence factor (see Melchers [8]).

$$\rho_{u,v} = \frac{\sum_{i=1}^{N} I_{u,i} I_{v,i}}{\sqrt{\sum_{i=1}^{N} I_{u,i}^2 \sum_{i=1}^{N} I_{v,i}^2}} \tag{10}$$

If the mean correlation is low, say less than 0.2, the probability of violation will be near the upper bound and if the correlation is high, say above 0.8, the probability of violation will be close to the lower bound (see Melchers [8]). For vertical tendons in Swedish containments the mean correlation is concluded to be low (se example in Section 7).

It is also possible to calculate more exact values of the probability of violation by using numerical methods (Monte Carlo simulation), which is made in the example in the Section 7.

6. Tendon dropout

Information concerning the probability of broken tendons, i.e. tendons which has lost its bearing capacity, is limited (damaged tendons is referred to as tendon dropout). Severe corrosion attacks have been detected on tendons in Swedish containments in a few cases. In these cases, the corrosion has been concluded to be caused by water in the tendon ducts. The Swedish power plant companies have after these incidents included the risk of tendon dropout in the evaluation of the prestress level. This evaluation has been made by assuming that one tendon in a certain part of the containment has lost its bearing capacity. An opinion about the general risk of tendon dropout could be found if an extensive survey of reported damaged tendons and remarks from inspection programs were made. This type of survey is not included in this paper, here it is assumed that a general probability of tendon dropout $P(\mathrm{TD})$ at the time of the in-service inspection is known (TD is defined as the event of a tendon dropout). The probability of dropout $P(\mathrm{TD})$ will have a dependency with time due to ageing mechanisms as corrosion. The risk of corrosion increases with time due to changes of the quality of the medium protecting the tendons or increased risk of water in the ducts due to cracks in the concrete wall.

Knowing $P(\mathrm{TD})$ at the time of the in-service inspection the probability of k tendon dropouts in a containment with totally N tendons can be calculated with the binominal distribution (expression 11)), where D_k represents the event of k tendons being defected.

$$P(D_k) = \binom{N}{k} P(\mathrm{TD})^k (1 - P(\mathrm{TD}))^{N-k} \tag{11}$$

The location of tendon dropouts in the structure will affect the probability of violation, especially for $k > 1$ where the distance between defected tendons will be important. The event D_k is therefore divided into r possible sub-events, where each sub-event represents one combination of tendon dropouts. Assuming that all sub-events have the same probability to occur the probability of violation conditional on k tendon dropouts can be calculated as

$$P(E|D_k) = \frac{1}{r} \sum_{i=1}^{r} P(E_{k,i}) \tag{12}$$

where $r = \binom{N}{k}$ and $P(E_{k,i})$ is the probability of violation for the ith combination of k tendon dropouts. Cracks in the concrete wall (causing corroded tendons) could be localised to some part of the containment and therefore affect several of adjacent tendons. This effect could be included in expression (12) by giving sub-events for tendon dropouts in adjacent tendons higher probability to occur. To be able to give more specific recommendations about the elevated probability of tendon dropout in adjacent tendons a more comprehensive study about reported damaged tendons have to be done.

The event of violation given k tendon dropouts $(E|D_k)$ and the event of k tendon dropouts (D_k) can be seen as independent events. The total probability of violation including the risk of tendon dropout can therefore be calculated as

$$P(E) = \sum_{k=0}^{N} P(E|D_k) P(D_k) \tag{13}$$

In practice the number of combinations r in expression (12) will be extensive for large number of tendon dropouts (k). With reasonable values on the general probability of tendon dropout $P(\mathrm{TD})$, the probability of a large number of tendon dropouts $(P(D_k))$ will be low. Expression (13) can therefore be truncated at $k = b$, where $P(D_b)$ is insignificant. The risk of tendon dropout is included in the analytical and numerical example in the next section.

7. Example: perimeter wall, vertical tendons

In this example, the vertical prestress in a containment wall is evaluated with the analytic method described above and with a numerical calculation (Monte Carlo simulation). The geometry of the structure is realistic, but the measured tendon force is chosen to fit this example.

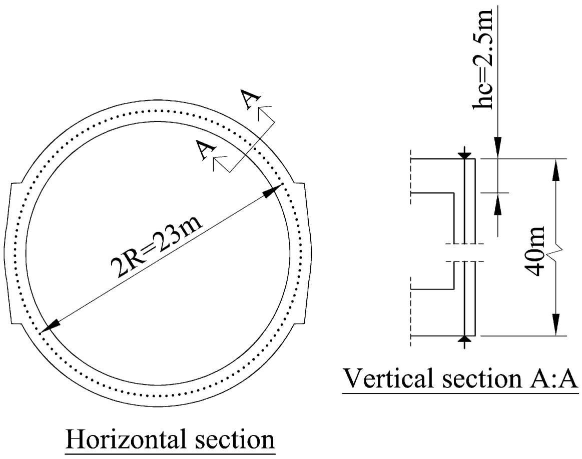

Fig. 7. Sketch of containment.

The containment consists of totally 141 VSL tendons (with 19 strands consisting of 7 wires each), which are located in the centre of the wall. The height of the containment is 40 m and the radius (wall centre) is 11.5 m (see Fig. 7). The mean value of the tendon force in this example is 1.8 MN (evaluated from 6 measured tendon forces at the current in-service inspection) and the standard deviation is 0.2 MN (evaluated from trends valid for containments with VSL tendons). The distance a between the tendons is 0.5 m ($a = 2\pi R/N$).

The division in structural parts is made according to Section 5 and for this particular example one structural part is chosen for each tendon (see Fig. 5). The prestress is checked in a location in the lower edge of the upper slab, i.e. 2.5 m below the top anchorage of the tendons (see Fig. 7). The prestress at this level is the sum of a number of influencing tendons according to expression (3). In this particular example, it is reasonable to approximate the influence factors by expression (14) (solution by Boussinesq, see Timoshenko and Goodier [9]). This expression describes the stresses in a semi-infinite elastic plate with concentrated load on the edge, which is assumed to be valid for this case

$$I_{ij} = \frac{2x^3}{\pi(x^2 + (y_i - y_j)^2)^2} \tag{14}$$

x represents the depth ($x = h_c = 2.5$ m) and $(y_i - y_j)$ the horizontal distance to the load (see Fig. 5). The calculated influence factors are shown in Table 2. The containment is more or less rotary-symmetric, so the sum of influence factors is not varying between different structural parts (i.e. $q_{R,j} = q_R$).

The required prestress per unit length of the containment wall q_S is calculated in expression (15), where the design pressure (p_d) for this containment is 0.5 MPa.

$$q_S = \frac{\pi R^2}{2\pi R} p_d = \frac{R}{2} p_d = 2.88 \text{ MN/m} \tag{15}$$

To include the effects of tendon dropout an assumption of the general probability of tendon dropout (P(TD)) is required (see Section 4). In this example, P(TD) is assumed to be 10^{-3} at the time of the inspection. The probability of different fallouts $P(D_k)$ is calculated by expression (11) and shown in Tables 3 and 4.

Table 2
Influence factors calculated according to expression (14) ($x = h_c = 2.5$ m)

$i - j$ (±)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$y_i - y_j$ (m)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5
I_{ij} (m^{-1})	0.255	0.235	0.189	0.138	0.095	0.064	0.043	0.029	0.020	0.014	0.010	0.007	0.006	0.004	0.003	0.003

Table 3
Result from analytic calculation

k	0	1	2	3	4	$P(E)$
$P(D_k)$	8.68×10^{-1}	1.23×10^{-1}	8.59×10^{-3}	3.98×10^{-4}	1.38×10^{-5}	
$P(E\|D_k)$[a]	1.28×10^{-8}	9.05×10^{-3}	8.28×10^{-2}	2.12×10^{-1}	3.76×10^{-1}	
$P(E\|D_k)P(D_k)$	1.11×10^{-8}	1.11×10^{-3}	7.11×10^{-4}	8.44×10^{-5}	5.17×10^{-6}	1.91×10^{-3}

[a] Upper bound.

Table 4
Results from numerical calculation, $n_s = 100{,}000$ simulations

k	0	1	2	3	4	$P(E)$
$P(D_k)$	8.68×10^{-1}	1.23×10^{-1}	8.59×10^{-3}	3.98×10^{-4}	1.38×10^{-5}	
$P(E\|D_k)$	0	5.06×10^{-3}	5.84×10^{-2}	1.50×10^{-1}	2.75×10^{-1}	
$P(E\|D_k)P(D_k)$	0	6.20×10^{-4}	5.01×10^{-4}	5.96×10^{-5}	3.79×10^{-6}	1.19×10^{-3}

7.1. Analytic calculation

The mean and standard deviation for the prestress (μ_q and σ_q) at the critical level can be calculated by expressions (4) and (5) and the influence factors in Table 2. Only the 15 nearest tendons (on both sides of element j) are included in the calculation below. (To reproduce the values in Table 3 all influence factors are required)

$$\mu_q = \mu_{q,j} = \mu_F \left(I_{jj} + 2\sum_{i=1}^{15} I_{ij} \right) = 1.8 \times 1.97 = 3.55 \text{ MN/m}$$

$$\sigma_q = \sigma_{q,j} = \sigma_F \sqrt{I_{jj}^2 + 2\sum_{i=1}^{15} I_{ij}^2} = 0.2 \times 0.56 = 0.112 \text{ MN/m}$$

The upper bound can be calculated with expression (7) and is shown in Table 3. The mean correlation is calculated to 0.08 in this example (expression (9) and (10)). The probability of violation is therefore expected to be close to the upper bound.

To find the total probability of violation the possibility of tendon dropout is included. To avoid extensive calculation work, expression (11) is truncated at 4 dropouts. Table 3 shows that the probability of 4 or more tendon dropouts is small in this example, $P(D_{k>4}) = 1 - P(D_{k\leqslant 4}) = 3.9 \times 10^{-7}$. The maximum truncation error will therefore be 3.9×10^{-7} and compared to the conditional probability (10^{-4}) this will have small effects on the final result.

The upper bound probability of violation for 1–4 tendon dropouts is calculated with expression (12) and the results are shown in Table 3. The final upper bound for the probability of violation ($P(E)$) is calculated with expression (13) and shown in Table 3. For the chosen example the effect of tendon dropout is shown to be significant.

7.2. Numerical simulation

In the simulation the tendon forces in the structure are chosen randomly from the normal distribution with the mean and standard deviation (μ_F and σ_F) given for this example. To find the concrete prestress, expression (3) together with the influence factors from expression (14) is used. The minimum prestress for all structural parts is then compared with q_S and if the prestress is below the limit, a failure is recorded and added to a counter n_f. This procedure is repeated for a large number of samples (n_s) and the probability of violation ($P(E)$) is finally calculated with expression (16).

$$P(E) = \frac{n_f}{n_s} \tag{16}$$

To include the effect of tendon dropout the probability of violation is calculated for each fallout of tendon dropouts. In the same procedure as described above, k randomly selected tendons are given zero force. The probability of violation for $k = 0$ to 4 tendon dropouts is given in Table 4.

The final probability of violation ($P(E)$) is calculated with expression (13) and is shown in Table 4. The probability of violation is slightly lower in the numerical calculation than the upper bound for the probability of violation in the analytical calculation. This is expected since the correlation between the different structural elements is included in the numerical calculation.

8. Conclusions

The main function of the prestressing system in reactor containments is to resist the tensile stresses in the concrete arising from the internal accident pressure. It is required that the prestress counterbalance the tensile stresses i.e. no tensile stresses are accepted in the containment structure during an accident. To verify that the required prestress is fulfilled the force in a number of tendons (around 4%) is measured in lift-off tests at regular in-service inspections. This paper presents a reliability-based model evaluating the level of prestress for reactor containments with unbonded prestressing tendons.

It is not possible to achieve a complete assurance concerning the prestress level. It is assumed that the sample mean from one in-service inspection is a good estimation of the mean force for the whole population of tendons. The standard deviation is found to be a more unreliable parameter and it is recommended to evaluate this parameter from trends over time and not from a single in-service inspection.

The tendon forces are in the presented model assumed to be normally distributed (1) and statistically independent random variables (2). (1) The assumption of normality are investigated using measurements from two Swedish containments were a lager number of tendons where tested 16 and 19 years after the initial tensioning. It is concluded that the normal distribution fits well to these measurements. (2) The stiffness for the concrete structure is large compared to the tendon stiffness. This implies that a change of force in a tendon has an insignificant effect on other tendons in the structure, which speaks for a statistically independence between tendons. However, some dependence can be expected due to locally high temperature (increased long time losses) and variation of concrete quality (within batches).

Several tendons influence the prestress level in a certain position of the containment. The tendons influencing one structural part of the containment could be considered as units in a parallel system. The required prestress shall be fulfilled in all parts of the containment. The model for the whole structure could therefore be considered as a series system of all the structural parts (parallel systems). The structural parts in the series system are concluded to be unequally correlated. This makes it difficult to calculate the exact probability of violation for the whole structure. In the example in this paper, the mean correlation for the structure is concluded to be low, inferring that the probability of violation can be estimated by an upper bound expression. The results from the example show that the analytic upper bound probability is only slightly above the numerical probability of violation. The small difference is expected due to the concluded small mean correlation.

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